OCR
Oxford Cambridge and RSA

Model Solutions

Accredited

AS Level Further Mathematics A
Y533 Mechanics
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A
- Scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1 A roundabout in a playground can be modeled as a horizontal circular platform with centre O. The roundabout is free to rotate about a vertical axis through O. A child sits without slipping on the roundabout at a horizontal distance of 1.5 m from O and completes one revolution in 2.4 seconds.

(i) Calculate the speed of the child. **[3]**

(ii) Find the magnitude and direction of the acceleration of the child. **[3]**

i. $\frac{2\pi}{w} = t$

$\frac{2\pi}{w} = 2.4$

$w = 2.618$

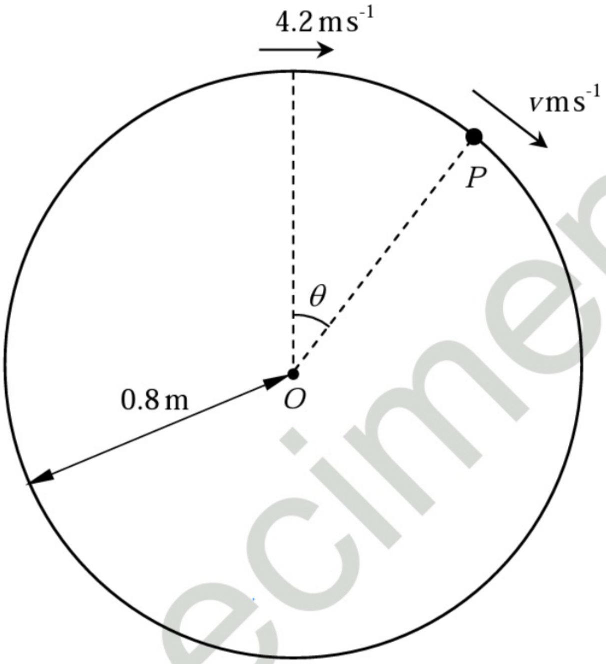
$v = rw = 1.5 \times 2.618 = 3.93 \text{ ms}^{-1}$ (3sf)

ii $a = \frac{v^2}{r} = \frac{\left(\frac{5}{4}\pi\right)^2}{1.5} = 10.3 \text{ ms}^{-2}$ (3sf)

Direction is towards the centre of the roundabout.

(towards centre O).

2



A smooth wire is shaped into a circle of centre O and radius 0.8 m. The wire is fixed in a vertical plane. A small bead P of mass 0.03 kg is threaded on the wire and is projected along the wire from the highest point with a speed of 4.2 m s^{-1}. When OP makes an angle θ with the upward vertical the speed of P is v m s^{-1} (see diagram).

Remember:

$KE = \frac{1}{2}mv^2 \qquad PE = mgh$

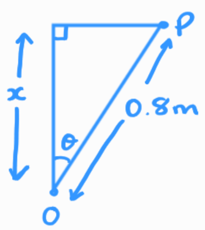
(i) Show that $v^2 = 33.32 - 15.68\cos\theta$. **[4]**

Initially: $KE = \frac{1}{2} \times 0.03 \times 4.2^2 = 0.2646$

$PE = 0$

Total energy $= 0.2646$

At angle θ: $KE = \frac{1}{2} \times 0.03 \times v^2 = 0.015v^2$ $\quad \cos\theta = \frac{x}{0.8}$



$PE = -0.03g(0.8 - 0.8\cos\theta)$

$= 0.2352(\cos\theta - 1)$

Total energy $= 0.015v^2 + 0.2352(\cos\theta - 1)$

By conservation of energy:

$0.2646 = 0.2352(\cos\theta - 1) + 0.015v^2$

$17.64 = 15.68\cos\theta - 15.68 + v^2$

$v^2 = 33.32 - 15.68\cos\theta$ (as required)

(ii) Prove that the bead is never at rest. **[1]**

(iii) Find the maximum value of v. **[2]**

ii. If $v = 0$, $\Rightarrow 0 = 33.32 - 15.68\cos\theta$

$\cos\theta = 2.125$

$2.125 > 1$ so this has no solutions, it can never be at rest.

iii. At max when $\cos\theta = -1$:

$v^2 = 33.32 - 15.68 \times -1$

$v^2 = 49$

$v = 7\ ms^{-1}$

3 **(i)** Write down the dimension of density. **[1]**

The workings of an oil pump consist of a right, solid cylinder which is partially submerged in oil. The cylinder is free to oscillate along its central axis which is vertical. If the base area of the pump is $0.4\ \text{m}^2$ and the density of the oil is $920\ \text{kg m}^{-3}$ then the period of oscillation of the pump is 0.7 s.
A student assumes that the period of oscillation of the pump is dependent only on the density of the oil, ρ, the acceleration due to gravity, g, and the surface area, A, of the circular base of the pump. The student attempts to test this assumption by stating that the period of oscillation, T, is given by $T = C\rho^\alpha g^\beta A^\gamma$ where C is a dimensionless constant.

(ii) Use dimensional analysis to find the values of α, β and γ. **[4]**

i. $\rho = ML^{-3}$

ii. $T = C\rho^\alpha g^\beta A^\gamma$

$T = (ML^{-3})^\alpha (LT^{-2})^\beta (L^2)^\gamma$

$T = M^\alpha L^{-3\alpha + \beta + 2\gamma} T^{-2\beta}$

M : $\alpha = 0$

T : $1 = -2\beta$

$\beta = -1/2$

L : $-3\alpha + \beta + 2\gamma = 0$

$0 - 1/2 + 2\gamma = 0$

$2\gamma = 1/2$

$\gamma = 1/4$

$\therefore \alpha = 0, \beta = -1/2, \gamma = 1/4$

(iii) Hence give the value of C to 3 significant figures. **[2]**

(iv) Comment, with justification, on the assumption made by the student that the formula for the period of oscillation of the pump was dependent on only ρ, g and A. **[2]**

iii. $T = C g^{-1/2} A^{1/4}$

sub in known values : $T = 0.7$ at $A = 0.4$

$0.7 = C g^{-1/2} A^{1/4}$

$0.7 = 0.254C$

$C = 2.76$

iv. $\alpha = 0$ so period of oscillation is not dependant on density in this model.

4 A car of mass 1250 kg experiences a resistance to its motion of magnitude kv^2 N, where k is a constant and v m s^{-1} is the car's speed. The car travels in a straight line along a horizontal road with its engine working at a constant rate of PW. At a point A on the road the car's speed is 15 m s^{-1} and it has an acceleration of magnitude 0.54 m s^{-2}. At a point B on the road the car's speed is 20 m s^{-1} and it has an acceleration of magnitude 0.3 m s^{-2}.

(i) Find the values of k and P. **[7]**

$$kv^2 \longleftarrow O \longrightarrow f$$

$$P = fv \Rightarrow f = \frac{P}{v}$$

At A : $f - kv^2 = ma$

$$\frac{P}{v} - k \times 15^2 = 1250 \times 0.54$$

$$\frac{P}{15} - 225k = 675$$

$$P - 3375k = 10125$$

At B : $f - kv^2 = ma$

$$\frac{P}{20} - k \times 20^2 = 1250 \times 0.3$$

$$P - 8000k = 7500$$

Setting these equations for P equal to each other:

$$10125 + 3375k = 7500 + 8000k$$

$$4625k = 2625$$

$$k = 0.568 \text{ (3sf)}$$

$$P = 7500 + 8000 \times 0.568\ldots$$

$$P = 12040.54054 \text{ W}$$

$$P = 12041 \text{ W}$$

The power is increased to 15 kW.

(ii) Calculate the maximum steady speed of the car on a straight horizontal road. **[3]**

Max speed happens when acceleration = 0

$F - kv^2 = 0$

$\frac{15000}{v} = 0.568v^2$

$v^3 = 26428.6$

$v = 29.8\ ms^{-1}$ (3sf)

5

The masses of two spheres A and B are $3m$ kg and m kg respectively. The spheres are moving towards each other with constant speeds $2u\text{ m s}^{-1}$ and $u\text{ m s}^{-1}$ respectively along the same straight line towards each other on a smooth horizontal surface (see diagram). The two spheres collide and the coefficient of restitution between the spheres is e. After colliding, A and B both move in the same direction with speeds $v\text{ m s}^{-1}$ and $w\text{ m s}^{-1}$, respectively.

(i) Find an expression for v in terms of e and u. **[6]**

Before: $\underset{3m}{O} \xrightarrow{2u}$ $\quad \xleftarrow{u} \underset{m}{O}$

After: $\underset{3m}{O} \xrightarrow{v}$ $\quad \underset{m}{O} \rightarrow w$

$e = \frac{w - v}{2u - -u}$

$e = \frac{w - v}{3u} \Rightarrow w = 3ue + v$

Momentum : $3m(2u) + m(-u) = 3m(v) + m(w)$

$6\cancel{m}u - \cancel{m}u = 3\cancel{m}v + \cancel{m}w$

$5u = 3v + w$

$5u = 3v + 3ue + v$

$4v = -3ue + 5u$

$v = \frac{u}{4}(5 - 3e)$

(ii) Write down unsimplified expressions in terms of e and u for

(a) the total kinetic energy of the spheres before the collision, **[1]**

(b) the total kinetic energy of the spheres after the collision. **[2]**

a. before $= \frac{1}{2}(3m)(2u)^2 + \frac{1}{2}(m)(-u)^2$

b. after $= \frac{1}{2}(3m)\left(\frac{u}{4}\right)^2(5-3e)^2 + \frac{1}{2}(m)\left(\frac{u}{4}\right)^2(5+9e)^2$

(iii) Given that the total kinetic energy of the spheres after the collision is λ times the total kinetic energy before the collision, show that

$$\lambda = \frac{27e^2 + 25}{52}.$$

[3]

$\frac{13}{2}\cancel{mu^2} \times \lambda = \frac{3}{32}\cancel{mu^2}(5-3e)^2 + \frac{1}{32}\cancel{mu^2}(5+9e)^2$

$208\lambda = 3(25 - 30e + 9e^2) + (25 + 90e + 81e^2)$

$208\lambda = 100 + 108e^2$

$\lambda = \frac{27e^2 + 25}{52}$

(iv) Comment on the cases when

(a) $\lambda = 1$,

(b) $\lambda = \frac{25}{52}$. **[3]**

a. $\lambda = 1 \Rightarrow 52 = 27e^2 + 25$

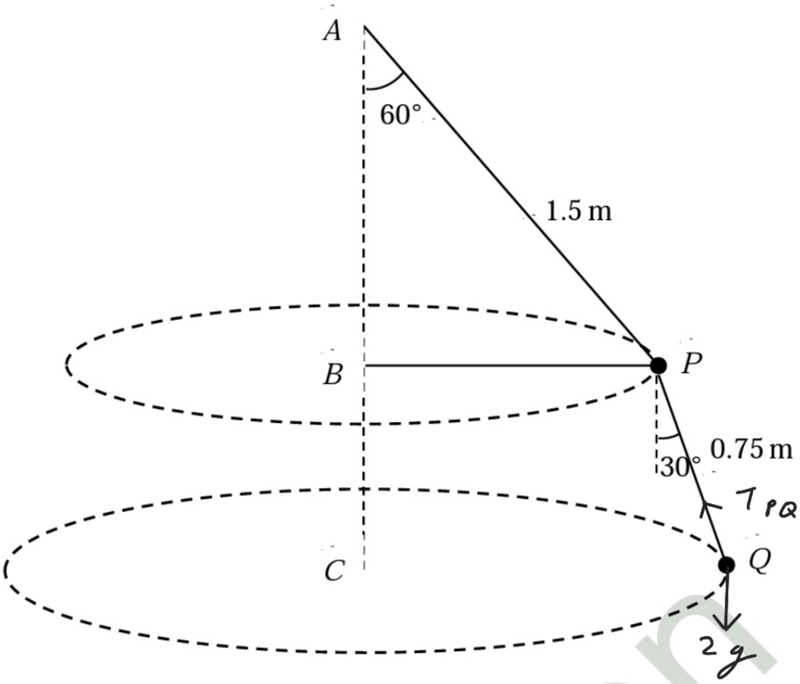
$e = 1$

e = 1 is a perfectly elastic collision, meaning no energy is lost.

b. $\lambda = \frac{25}{52} \Rightarrow e = 0$

The collision is perfectly inelastic, the particles join together.

6



The fixed points A, B and C are in a vertical line with A above B and B above C. A particle P of mass 2.5 kg is joined to A, to B and to a particle Q of mass 2 kg, by three light rods where the length of rod AP is 1.5 m and the length of rod PQ is 0.75 m. Particle P moves in a horizontal circle with centre B. Particle Q moves in a horizontal circle with centre C at the same constant angular speed ω as P, in such a way that A, B, P and Q are coplanar. The rod AP makes an angle of 60° with the downward vertical, rod PQ makes an angle of 30° with the downward vertical and rod BP is horizontal (see diagram).

(i) Find the tension in the rod PQ. **[2]**

(ii) Find ω. **[3]**

i. Resolve vertically : $T_{PQ} \cos 30 = 2g$

$T_{PQ} = 22.6\,N$

ii. $r = 1.5 \sin 60 + 0.75 \sin 30$

$r = 1.674$

$T_{PQ} \sin 30 = 2 r w^2$

$w^2 = \dfrac{22.6 \sin 30}{2 \times 1.674}$

$w^2 = 3.375$

$w = 1.84 \text{ rad s}^{-1}$

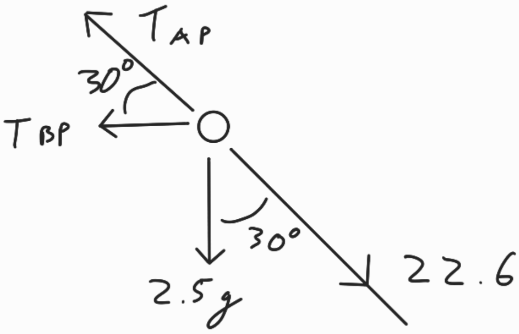
(iii) Find the speed of P. **[1]**

(iv) Find the tension in the rod AP. **[3]**

iii. $v_P = 1.5 \sin 60 \times 1.84$

$= 2.39 \text{ ms}^{-1}$

iv.



Resolve vertically: $T_{AP} \sin 30 = 2.5g + 22.6 \cos 30$

$\frac{1}{2} T_{AP} = 44.1$

$T_{AP} = 88.2 \text{ N}$

(v) Hence find the magnitude of the force in rod BP.
Decide whether this rod is under tension or compression. **[4]**

Resolve horizontally: $T_{BP} + T_{AP} \cos 30 - 22.6 \sin 30 = 2.5 \times (1.5 \sin 60) \omega^2$

$T_{BP} + 88.2 \cos 30 - 11.3 = 2.5 \times 1.299 \times 3.375$

$T_{BP} = -54.1 \text{ N}$

Magnitude $= 54.1$

$-54.1 < 0$ so the rod BP is under compression.